KING ABDULAZIZ UNIVERSITY
FACULTY OF SCIENCE
DEPARTMENT OF MATHEMATICS

ENTRANCE TEST FOR M.SC. MATHEMATICS

19th Jumada Awal, 1434 / 31st March 2013

Maximum Time: Three Hours

*Name:*____

I.D. #: ____

INSTRUCTIONS:
1. Solve all Eight questions given in the exam.
2. All questions are equally weighted.
3. Write complete steps in a proof.
4. To disprove a statement, provide a counter example.

Grading

	Assigned	Obtained
Q.1	(10)	
Q.2	(10)	
Q.3	(10)	
Q.4	(10)	
Q.5(A/B)	(10)	
Q.6	(10)	
Q.7	(10)	
Q.8	(10)	
Total:	(80)	

Q.1.

Multiple Choice Question:
In the following, cross the the circle ◯ for a correct statement.

(**A**) The function $f(x) = |cos x|$ is:
◯ continuous everywhere but not differentiable everywhere,
◯ not continuous at $\frac{(2k+1)\pi}{2}$, k is an integer,
◯ continuous everywhere but not differentiable at $\frac{(2k+1)\pi}{2}$,
◯ none of above.

(**B**) The quadratic function $f(x) = (x+3)^2$, $x \in [-3, 0]$ has:
◯ a maximum but not minimum on $[-3, 0]$,
◯ a minimum but not maximum on [-3,0],
◯ both maximum and minimum on [-3,0],
◯ neither maximum nor minimum on [-3,0].

(**C**) The function $f(x) = \cos x$ is one to one and onto if the domain and range are, respectively, specified as:
◯ $(-\infty, \infty)$, $[-1, 1]$,
◯ $[0, 2\pi]$, $[-1, 1]$,
◯ $[0, \pi]$, $[-1, 1]$,
◯ $[0, \frac{\pi}{2}]$, $[-1, 1]$.

(**D**) The equation $x^2 - y^2 = 0$:
◯ represents a hyperbola,
◯ represents a parabola,
◯ represents two intersecting lines,
◯ none of above.

(**E**) The set $P = \{(x, y, z) \mid ax + by + cz = 0;\ a, b, c \in \mathbb{R}\}$ is a:
◯ line of dimension one in space,
◯ line of dimension two in space,
◯ plane of dimension three in space,
◯ plane of dimension two in space.

Q.2.

($\mathbf{a}_1$) Write the linear transformation T corresponding to the matrix

$$B = \begin{bmatrix} 2 & 3 \\ 3 & -6 \end{bmatrix}.$$

($\mathbf{a}_2$) Compute the eigen values of T.

Q.2.
(**b**) Determine whether the following matrix is invertible:

$$A = \begin{bmatrix} 1 & 5 & 0 \\ 2 & 4 & -1 \\ 0 & -2 & 0 \end{bmatrix}.$$

Q.3.

(**a**) Let $H = \{1, -1, i, -i\}$ be the group of four complex numbers under multiplication.

True or False:

(1) H is cyclic ◯

(2) $(i)^{-1} = i$ ◯.

(**b**) If G is a group such that $(ab)^2 = a^2b^2$ for all $a, b \in G$; then show that G must be abelian.

Q.3.

(**c**) Define the center $Z(G)$ of a group G. Prove that $Z(G)$ is a subgroup of G.

Q.4.

(**a**) True or False:

Let F be a field. Then:

(1) $\{0\}$ is a maximal ideal of F..........◯.

(2) F itself is a mximal ideal of F.........◯.

(**b**) If R is a ring and U is an ideal of R; let

$$r(U) = \{x \in R : xu = 0, \forall u \in U\}.$$

Prove that $r(U)$ is an ideal of R.

Q.4.

(**c**) If R is a ring with the identity element 1 and ϕ is a homomorphism of R onto another ring S, then prove that $\phi(1)$ is the identity element of S.

Solve either Q.5A or Q.5B.

Q.5A.

(**a**) Define a bounded sequence and show that every convergent sequence of real numbers is bounded.

(**b**) Give an example of a bounded sequence which is not convergent.

(**c**) Give an example of a bounded increasing sequence which is convergent.

If you have solved Q.5A then do not solve Q.5B.

Q.5B.

(a) If f is analytic in a region and if $|f|$ is constant there, prove that f is constant.

(b) Show that

$$\sum_{n=0}^{\infty} \frac{z^n}{n!}$$

converges for all z.

Q.6.

Define $\mathcal{T} \subset \mathcal{P}(\mathbb{R})$ as follows: A subset U of $\mathbb{R}$ is in $\mathcal{T}$ if and only if either $U = \emptyset$ or $\sqrt{2} \in U$. That is,

$$\tau \ = \{\, \emptyset \,\} \cup \{\, U \subseteq \mathbb{R} \,:\, \sqrt{2} \in U \,\}.$$

Answer the following problems:

(a) Prove that $\mathcal{T}$ is a topology on $\mathbb{R}$. This topology is called the *particular point topology.*

(b) Calculate in $(\,\mathbb{R}\,, \mathcal{T}\,)$ each of the following:

(1) $\mathrm{int}[0,3]$.

(2) $\overline{(1,2)}$.

(c) Prove or disprove: $(\mathbb{R}, \mathcal{T})$ is T_2.

(d) Prove or disprove: $(\mathbb{R}, \mathcal{T})$ is separable.

Q.7.
(**a**) Find a particular solution of:

$$u_{xx} + u_{yy} = 0, \quad u(0, y) = u(L, y) = 0, \ \ u(x, 0) = 0, u(x, L) = 1.$$

Q.7.
(**b**) Find a general solution of:

$$(y - x + 1)y' = 1.$$

Q.8.

Use Newton's method to approximate the root of the nonlinear equation

$$e^{-x} - x = 0, \ x_0 = 0.5.$$

Carry out two iterations only.